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EFFECT OF CONDUCTION IN WALL ON HEAT TRANSFER WITH TURBULENT FLOW BETWEEN PARALLEL PLATES

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Abstract—Wall conduction effects on steady-state turbulent flow heat-transfer experiments are examined, and an analysis of heat transfer with axial conduction in the wall bounding a fluid with turbulent flow is developed to determine the effects of conduction in the wall on heat transfer with turbulent flow between parallel plates.

From the results of numerical calculation for Reynolds numbers in the range $10^4 \leqslant Re \leqslant 10^5$ and for Prandtl numbers in the range $0.01 \leqslant Pr \leqslant 10$, it was confirmed that the ratio of thermal conductivity of wall to that of fluid and the thickness of wall could have significant effects on heat transfer and temperature field in the fluid adjacent to the wall. Experiments on the flat plate were conducted and the experimental results are in good agreement with the analytical results.

NOMENCLATURE

a_i, $(i+1)\tau_{i+1}$;
b, plate thickness [m];
b^*, b/y_0;
C_m, coefficient in equation (6);
Gz, Graetz number $= 4y_0 Re Pr/L$;
H_m, $C_m R'_m(0)$;
K, k_s/k_f;
k, thermal conductivity [W/m·K];
L, length of heat-transfer section of plate [m];
Nu, local Nusselt number;
Nu_F, local Nusselt number based on constant heat flux at solid–fluid interface;
Nu_T, local Nusselt number based on constant temperature at solid–fluid interface;
Nu_∞, fully developed Nusselt number;
P_m, $16\lambda_m^2/Gz$;
Pr, Prandtl number;
Q, dimensionless temperature gradient;
q, heat flux [W/m²];
R, R_m, eigenfunction;
Re, Reynolds number;
t, temperature [K];
u, velocity of fluid [m/s];
u^+, $u/\sqrt{(\tau_w/\rho)}$;
u^*, $u/V = u^+/V^+$;
V, mean velocity of fluid [m/s];
V^+, $V/\sqrt{(\tau_w/\rho)}$;
x, coordinate parallel to flat plate [m];
x^*, x/L;
y, coordinate normal to flat plate [m];
y^+, $y\sqrt{(\tau_w/\rho)}/\nu$;
y^*, $y/y_0 = y^+/y_0^+$;
y_0, half width of duct [m].

Greek symbols

α, thermal diffusivity [m²/s];
β, coefficient defined in equation (22);
γ, $1+Pr(\varepsilon_h/\nu)$;
δ, coefficient defined in equation (23);
ε_h, eddy diffusivity for heat [m²/s];
ε_v, eddy diffusivity for momentum [m²/s];
θ, dimensionless temperature defined by equation (5);
Θ, dimensionless temperature defined by equation (13);
ν, kinematic viscosity of fluid [m²/s];
λ, λ_m, eigenvalue;
ξ, dummy variable;
ρ, density of fluid [kg/m³];
τ_i, coefficient in equation (15);
τ_w, shear stress at interface [N/m²].

Subscripts

e, entrance;
f, fluid;
o, solid–fluid interface;
s, solid;
w, lower surface of plate.

INTRODUCTION

IN MOST previous analyses on the heat transfer with forced convective flow, it is common practice to prescribe the temperature, the heat flux, or a combination of the two at the solid–fluid interface. In most real cases, however, these boundary conditions cannot be known *a priori*, but depend on the coupled mechanism of heat transfer in the fluid and conduction in the solid.

It is then necessary to solve the energy equations for the fluid and the solid body simultaneously under the conditions of continuity in the heat flux and temperature at the interface. For example, in the design and analysis of a heat exchanger, axial conduction of heat in the wall bounding a fluid is usually ignored, but it can have significant effects on the heat transfer and temperature field in the fluid adjacent to the wall. This is especially true in the thermal entrance region.

Recently, Perelman [1] called this type of problem a "conjugated" boundary value problem, and presented the model of slip flow around a body with distributed heat sources, but no numerical results were given.

Sell and Hudson [2] treated the problem of heat transfer to a slug flow past a flat plate and Davis *et al.* [3] studied the effect of conduction in the wall on the heat transfer to Poiseulle–Couette flow between parallel plates. Olsson [4] considered the problem of heat transfer to a finite wedge-shaped fin in laminar flow. Luikov *et al.* [5, 6] carried out a series of analyses on these problems in laminar flow. The previous published papers deal with the problem of heat transfer only in laminar flow, and we have few analytical treatments for turbulent flow.

From a practical point of view, however, it is often desirable to use short passages with turbulent flow in a heat exchanger in order to take advantage of the high heat-transfer coefficients in the entrance region.

The purpose of the present study is to analyse a conjugated heat-transfer problem with turbulent flow between parallel plates. The problem deals with heat transfer between a flat plate of finite thickness and the fluid in contact with it. The boundary conditions of the lower surface of the plate that contacts a heat source are taken to be the uniform heat flux or the constant temperature. The main objective is to determine how the interfacial temperature and the local Nusselt number variations (that are influenced by axial conduction in the wall) can be predicted from solutions for the simultaneous energy equations for both fluid phase and solid wall, and to interpret the results of our experimental works.

1. MATHEMATICAL ANALYSIS

1.1. *Derivation of fundamental equations*

Consider turbulent flow between parallel plates, shown schematically in Fig. 1. In the present work,

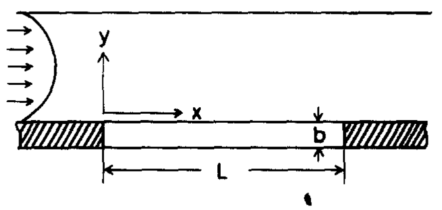

FIG. 1. Schematic model of heat transfer section of duct.

axial conduction in the fluid and viscous heat dissipation can be ignored and the physical properties of the fluid are assumed to be constant. The temperature field in the fluid is described by

$$u\frac{\partial t_f}{\partial x} = \frac{\partial}{\partial y}\left[(\alpha+\varepsilon_h)\frac{\partial t_f}{\partial y}\right] \tag{1}$$

Boundary conditions

$$\begin{aligned} &x \leqslant 0: t_f = t_{fe} \\ &x > 0: y = 0, \quad t_f = t_{fo}(x) \\ &\quad : y = 2y_0, \quad \partial t_f/\partial y = 0. \end{aligned} \tag{2}$$

The temperature field in the wall bounding the fluid is given by

$$\frac{\partial^2 t_s}{\partial x^2}+\frac{\partial^2 t_s}{\partial y^2} = 0 \tag{3}$$

Boundary conditions

$x = 0, L : \partial t_s/\partial x = 0$

$y = 0 \quad : t_s = t_{so}(x)$

$y = -b : \partial t_s/\partial y = -q_w/k_s$ (for the constant heat flux at the lower surface of the plate)

or $\quad : t_s = t_w$ (for the constant temperature at the lower surface of the plate). (4)

1.2. *Analysis of energy equation for the fluid*

The temperature can be written in terms of the solution for a constant interfacial temperature by applying the extended Duhamel's theorem.

First, we solve the problem with a constant temperature condition (uniform temperature on one wall, the other wall insulated). The dimensionless variables are defined as

$$u^* = \frac{u}{v} = \frac{u^+}{V^+}, \quad x^* = \frac{x}{L},$$
$$y^* = \frac{y}{y_0} = \frac{y^+}{y_0^+}, \quad \theta_f = \frac{t_{fo}-t_f}{t_{fo}-t_{fe}}. \tag{5}$$

We obtain by the method of separation of variables

$$\theta_f = \sum_{m=0}^{\infty} C_m R_m(y^*)\exp\left(-\frac{16}{Gz}\lambda_m^2 x^*\right) \tag{6}$$

where λ_m, R_m are the eigenvalues and eigenfunctions of the Sturm–Liouville problem

$$\frac{\mathrm{d}}{\mathrm{d}y^*}\left(\gamma\frac{\mathrm{d}R}{\mathrm{d}y^*}\right)+\lambda^2 u^* R = 0 \tag{7}$$

where

$$\gamma = 1+Pr\frac{\varepsilon_h}{\nu}$$

Boundary conditions

$$\begin{aligned} &y^* = 0 : R = 0 \\ &y^* = 2 : \mathrm{d}R/\mathrm{d}y^* = 0. \end{aligned} \tag{8}$$

The coefficients C_m are given by

$$C_m = \frac{\int_0^2 u^* R_m(y^*)\,\mathrm{d}y^*}{\int_0^2 u^* R_m^2(y^*)\,\mathrm{d}y^*}. \tag{9}$$

The mixed-mean temperature and the local Nusselt number are given by

$$\theta_{fm} = \tfrac{1}{2}\sum_{m=0}^{\infty}\frac{C_m R_m'(0)}{\lambda_m^2}\exp(-16\lambda_m^2 x^*/Gz) \tag{10}$$

$$Nu = \frac{8\sum_{m=0}^{\infty} C_m R_m'(0)\exp(-16\lambda_m^2 x^*/Gz)}{\sum_{m=0}^{\infty}\frac{C_m R_m'(0)}{\lambda_m^2}\exp(-16\lambda_m^2 x^*/Gz)}. \tag{11}$$

Hatton *et al.* [7] have tabulated the eigenvalues and the constants for this problem for the various values of Prandtl number and Reynolds number. However, there are ambiguous statements in the eddy diffusivity distribution and the fluid velocity distribution. They assumed that the ratio of the eddy diffusivities for momentum and heat was unity. To obtain the more accurate solution, we used the eddy diffusivity and the fluid velocity distribution presented by Mizushina *et al.* [8, 9] from a reconsideration of the available experimental works (cf. Appendix 1). We used the turbulent Prandtl number as follows. ($0 \leqslant y^* \leqslant 1$)

$Pr = 0.01$

$= 0.1$:

$$\varepsilon_v/\varepsilon_h = \frac{[1+90Pr^{3/2}(\varepsilon_v/v)^{1/4}][35+(\varepsilon_v/v)]}{[0.025Pr(\varepsilon_v/v)+90Pr^{3/2}(\varepsilon_v/v)^{1/4}][45+(\varepsilon_v/v)]}$$

(Notter *et al.*) [10]

$Pr = 0.7 : \varepsilon_v/\varepsilon_h = 0.86$ (Larson *et al.*) [11]

$Pr = 10 \quad \varepsilon_v/\varepsilon_h = 1/(0.1265y^*+1.064)$

(Gowen *et al.*) [12]. (12)

New dimensionless variables are defined by

$\Theta = t/t_{fe}$ (for the constant heat flux at the lower surface of the plate)

$\Theta = (t-t_w)/(t_{fe}-t_w)$ (for the constant temperature at the lower surface of the plate). (13)

The solution for the problem with a variable interfacial temperature can be written by applying Duhamel's theorem to equations (6) and (13) to give

$$\Theta_f = \frac{\partial}{\partial x^*}\int_0^{x^*}\left\{\Theta_{fo}(\xi)-[\Theta_{fo}(\xi)-1]\sum_{m=0}^{\infty}C_m R_m(y^*)\times\exp[-P_m(x^*-\xi)]\right\}\mathrm{d}\xi \quad (14)$$

where, it is assumed that the interfacial temperature is given by

$$\Theta_{fo}(\xi) = 1+\tau_0+\tau_1\xi+\tau_2\xi^2+\ldots+\tau_i\xi^i+\ldots. \quad (15)$$

The dimensionless temperature gradient at the interface is given by

$$Q_f = -\left(\frac{\mathrm{d}\Theta_f}{\mathrm{d}y^*}\right)_{y^*=0} = \tau_0\sum_{m=0}^{\infty}H_m\,\mathrm{e}^{-P_m x^*}+a_0\sum_{m=0}^{\infty}\frac{H_m}{P_m}(1-\mathrm{e}^{-P_m x^*})+\ldots + a_i\sum_{m=0}^{\infty}\frac{H_m}{P_m}\left[x^{*i}-\frac{i}{P_m}\left[x^{*(i-1)}-\frac{(i-1)}{P_m}\left[\ldots -\frac{1}{P_m}(1-\mathrm{e}^{-P_m x^*})\ldots\right]\right]\right]+\ldots \quad (16)$$

where

$Q_f = q_f y_0/k_f t_{fe}$ (for the constant heat flux at the lower surface of the plate)

$Q_f = q_f y_0/k_f(t_{fe}-t_w)$ (for the constant temperature at the lower surface of the plate)

$P_m = 16\lambda_m^2/Gz,\ H_m = C_m R_m'(0),\ a_i = (i+1)\tau_{i+1}$.

1.3. *Analysis of energy equation for the solid*

Equations (3) and (4) (the equation of heat conduction in the wall) are transformed into dimensionless form as those for the fluid. The solutions are given by equations (17) and (18) for the constant heat flux, and by equations (19) and (20) for the constant temperature at the lower surface.

$$\Theta_s = -Q_w y^* + \int_0^1 \Theta_{so}(x^*)\,\mathrm{d}x^* + 2\sum_{n=1}^{\infty}\frac{\cosh[n\pi y_0(y^*+b^*)/L]}{\cosh[n\pi y_0 b^*/L]}\cos n\pi x^* \times\int_0^1\Theta_{so}(x^*)\cos n\pi x^*\,\mathrm{d}x^* \quad (17)$$

$$Q_s = -\left(\frac{\mathrm{d}\Theta_s}{\mathrm{d}y^*}\right)_{y^*=0} = Q_w - 2\sum_{n=1}^{\infty}\left(\frac{n\pi y_0}{L}\right)\tanh\left(\frac{n\pi y_0}{L}b^*\right)\cos n\pi x^* \times\int_0^1\Theta_{so}(x^*)\cos n\pi x^*\,\mathrm{d}x^* \quad (18)$$

where

$Q_s = q_s y_0/k_s t_{fe},\quad Q_w = q_w y_0/k_s t_{fe}$

$$\Theta_s = \left(\frac{y^*}{b^*}+1\right)\int_0^1\Theta_{so}(x^*)\,\mathrm{d}x^* + 2\sum_{n=1}^{\infty}\frac{\sinh[n\pi y_0(y^*+b^*)/L]}{\sinh[n\pi y_0 b^*/L]}\cos n\pi x^* \times\int_0^1\Theta_{so}(x^*)\cos n\pi x^*\,\mathrm{d}x^* \quad (19)$$

$$Q_s = -\left(\frac{\mathrm{d}\Theta_s}{\mathrm{d}y^*}\right)_{y^*=0} = -\frac{1}{b^*}\int_0^1\Theta_{so}(x^*)\,\mathrm{d}x^* - 2\sum_{n=1}^{\infty}\left(\frac{n\pi y_0}{L}\right)\coth\left(\frac{n\pi y_0}{L}b^*\right)\cos n\pi x^* \times\int_0^1\Theta_{so}(x^*)\cos n\pi x^*\,\mathrm{d}x^* \quad (20)$$

where

$Q_s = q_s y_0/k_s(t_{fe}-t_w),\quad \Theta_{so}(x^*) = 1+\sum_{i=0}^{\infty}\tau_i x^{*i}$.

1.4. *Heat exchange between the fluid and the solid wall*

The following relations are applicable at the interface

$$\Theta_{fo} = \Theta_{so}, \qquad Q_f = KQ_s \quad (21)$$

where $K = k_s/k_f$.

In order to determine the coefficients $\tau_0, \tau_1, \tau_2, \ldots$ to obtain the interfacial temperature distribution, the orthogonality of cosine function is applied to equation (21) (Sakakibara *et al.*) [13, 14]. A system of linear simultaneous equations which can be solved to evaluate the coefficients is obtained by

$$\int_0^1\sum_{m=0}^{\infty}\left[\frac{H_m}{P_m}\left\{a_0\,\mathrm{e}^{-P_m x^*}-\sum_{i=0}^{\infty}a_i x^{*i}+\sum_{i=1}^{\infty}ia_iF_{i-1}(x^*)\right\} - H_m\tau_0\,\mathrm{e}^{-P_m x^*}\right]\mathrm{d}x^* = K\beta \quad (22)$$

$$\int_0^1 \sum_{m=0}^{\infty}\left[\frac{H_m}{P_m}\left\{a_0 e^{-P_m x^*} - \sum_{i=0}^{\infty} a_i x^{*i} + \sum_{i=1}^{\infty} i a_i F_{i-1}(x^*)\right\} - H_m \tau_0 e^{-P_m x^*}\right]\cos n\pi x^* \, dx^*$$

$$= \frac{Kn\pi y_0}{L}\delta \int_0^1 \left(1+\sum_{i=0}^{\infty} \tau_i x^{*i}\right)\cos n\pi x^* \, dx^* \quad (23)$$

where

$$F_0(x^*) = \{1-e^{-P_m x^*}\}/P_m$$
$$F_i(x^*) = \{x^{*i} - iF_{i-1}(x^*)\}/P_m.$$

For the constant heat flux at the lower surface of the flat plate

$$\beta = -Q_w, \quad \delta = \tanh\left(\frac{n\pi y_0 b^*}{L}\right)$$

and for the constant temperature at the lower surface of the flat plate

$$\beta = \frac{1}{b^*}\int_0^1 \int_0^1 \left(1+\sum_{i=0}^{\infty} \tau_i x^{*i}\right) dx^* \, dx^*$$

$$\delta = \coth\left(\frac{n\pi y_0 b^*}{L}\right)\dagger,$$

the mixed mean temperature and the local Nusselt number are given

$$\Theta_{fm} = 1 + \frac{8}{Gz}\int_0^{x^*} Q_f \, dx^* \quad (24)$$

$$Nu = \frac{4Q_f}{(\Theta_{fo} - \Theta_{fm})}. \quad (25)$$

The local Nusselt number is expressed by

$$Nu = \frac{Gz}{2}\left[\left(\tau_1 \cdot \frac{Gz}{8} + \tau_2 \cdot \frac{Gz}{4} x^* + \tau_3 \cdot \frac{3Gz}{8} x^{*2} + \ldots\right) + \sum_{m=0}^{\infty} H_m \left\{\tau_0 e^{-P_m x^*} - \frac{\tau_1}{P_m} e^{-P_m x^*} - \frac{2\tau_2}{P_m^2}(1-e^{-P_m x^*}) - \frac{6\tau_3}{P_m^2}\left(x^* - \frac{1}{P_m} + \frac{1}{P_m} e^{-P_m x^*}\right) - \ldots\right\}\right]$$
$$\Bigg/ \sum_{m=0}^{\infty} H_m \left\{\frac{\tau_0}{P_m} e^{-P_m x^*} + \frac{\tau_1}{P_m^2}(1-e^{-P_m x^*}) + \frac{2\tau_2}{P_m^2}\left(x^* - \frac{1}{P_m} + \frac{1}{P_m} e^{-P_m x^*}\right) + \frac{6\tau_3}{P_m^2}\left(\frac{x^{*2}}{2} - \frac{x^*}{P_m} + \frac{1}{P_m^2} - \frac{1}{P_m^2} e^{-P_m x^*}\right) + \ldots\right\}. \quad (26)$$

2. ANALYTICAL SOLUTIONS AND CONSIDERATIONS

2.1. *Solution of eigenvalue problem*

The values of λ_m and $C_m R_m'(0)$ for three Reynolds numbers and four Prandtl numbers are given to twenty terms in Table 1. In Fig. 2, our solution for unsymmetrical heat transfer is compared with Hatton *et al.*'s solution [7] to the analogous energy equation for turbulent flow between parallel plates. For $Pr = 0.1$, our solutions are in fairly good agreement with Hatton *et al.*'s solutions. On the other hand, our solutions show lower Nusselt numbers than that obtained by Hatton *et al.*'s solutions for $Pr = 10$. This may be attributed to the difference of the values of the eddy diffusivity, the fluid velocity distribution and the turbulent Prandtl number used in solving the energy equation. Hatton *et al.*'s solution is also limited to values of $x/4y_0$ greater than 1. However, our solution described here is effective to values of $x/4y_0$ greater than 0.1.

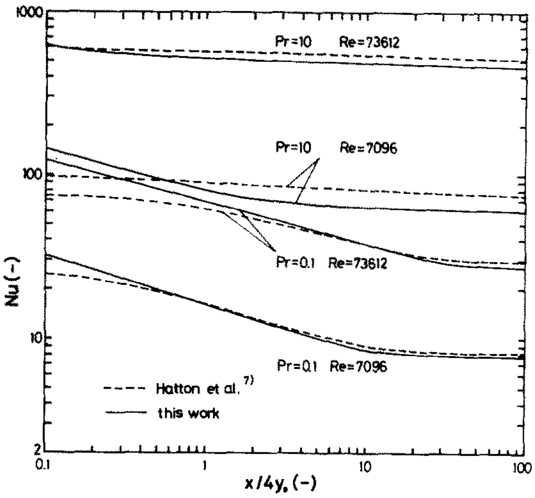

FIG. 2. Comparison of present numerical values with those of Hatton *et al.* (uniform temperature on one side, the other side insulated).

2.2. *Solution of the conjugated problem*

In the previous papers, the effects of axial conduction in the wall on the interfacial temperature were estimated by Sell *et al.* [2] and Davis *et al.* [3] for the constant heat flux at the lower plate surface. However, the general conclusion was not found for the local Nusselt number distribution. In this work, we made the numerical analysis for the effect of wall conduction on the local Nusselt number and the interfacial temperature distribution. The effect of wall conduction on the heat transfer was also discussed for a flat plate with the constant heat flux and the constant temperature at the lower plate surface contacting the heat source.

In the calculation, the orthogonality of cosine function is applied to equation (21). The linear simultaneous equations, equations (22) and (23), have a unique solution and were solved by Gauss–Jordan method. Equation (15) was found to be a rapidly convergent series, so the results presented here were calculated by third order polynomials. The eigenvalues and constants in equations (22) and (23) were estimated by the first twenty terms.

The equations describing the turbulent conjugated heat transfer are solved numerically for $10^4 \leqslant Re \leqslant 10^5$, $0.01 \leqslant Pr \leqslant 10$, $y_0/L = 0.02$, $0.001 \leqslant b/L \leqslant 0.05$ and $1 \leqslant K \leqslant 50000$. Figures 3–6 show the representative

†The notation of δ in *Kagaku Kōgaku* [13] should be read $\delta = \coth n\pi b$. It is corrected in *Heat Transfer, Japan. Res.* [13].

Table 1. Eigenvalues and constants. Uniform temperature on one side, the other side insulated

m	$Pr = 0.01$ λ_m	$Pr = 0.01$ $C_m R'_m(0)$	$Pr = 0.1$ λ_m	$Pr = 0.1$ $C_m R'_m(0)$	$Pr = 0.7$ λ_m	$Pr = 0.7$ $C_m R'_m(0)$	$Pr = 10$ λ_m	$Pr = 10$ $C_m R'_m(0)$
$Re = 10000$								
0	0.7907424	1.062687	1.068251	2.023355	1.800767	6.214894	3.200927	20.42286
1	2.387000	0.9141356	3.274810	1.495191	6.222992	2.343795	19.76490	1.577115
2	3.980757	0.8649360	5.509067	1.182879	11.00993	1.213341	36.94320	0.7824505
3	5.576050	0.8223678	7.766698	0.9583012	15.83777	0.8386898	53.26427	0.7370211
4	7.173212	0.7807557	10.02954	0.8155993	20.50739	0.7250058	66.34003	0.8934897
5	8.772061	0.7404411	12.28273	0.7333593	24.91433	0.7319784	77.20027	1.670051
6	10.37223	0.7031015	14.52004	0.6948843	29.12993	0.8561895	88.83077	2.942503
7	11.97332	0.6691142	16.74341	0.6837522	33.32843	1.058112	101.3196	2.601044
8	13.57503	0.6388302	18.95881	0.6879798	37.57233	1.241457	114.9463	1.847579
9	15.17713	0.6118909	21.17203	0.6957883	41.86729	1.273128	128.4029	1.623957
10	16.77947	0.5877954	23.38667	0.6985316	46.22904	1.173910	141.1888	1.835968
11	18.38203	0.5662184	25.60425	0.6944153	50.64224	1.050001	153.8608	2.210371
12	19.98471	0.5468987	27.82464	0.6834765	55.07477	0.9498438	166.7016	2.325579
13	21.58744	0.5295461	30.04692	0.6702539	59.50155	0.8877637	179.9056	2.054597
14	23.19011	0.5140634	32.27043	0.6545382	63.91024	0.8581674	193.3113	1.800787
15	24.79267	0.5002540	34.49548	0.6370509	68.29502	0.8559606	206.6267	1.698895
16	26.39505	0.4879510	36.72321	0.6157694	72.65587	0.8827127	219.7017	1.767534
17	27.99724	0.4769195	38.95478	0.5909196	77.00084	0.9225720	232.6632	1.928840
18	29.59928	0.4668772	41.19025	0.5649742	81.34468	0.9539520	245.6602	1.990960
19	31.20112	0.4576459	43.42787	0.5408628	85.70704	0.9427588	258.8776	1.848189
$Re = 50000$								
0	0.8184341	1.127200	1.635171	4.801148	3.320573	21.21170	6.408935	81.69296
1	2.466011	0.9868990	5.056098	3.307465	11.65080	7.426775	38.77928	6.063249
2	4.110855	0.9463364	8.554299	2.432236	20.73845	3.616122	72.88529	2.336639
3	5.756289	0.9147396	12.09812	1.890018	29.95922	2.361603	107.0561	1.530405
4	7.402283	0.8871595	15.63778	1.592916	39.02563	1.888524	139.8280	1.311778
5	9.048500	0.8637160	19.14322	1.452925	47.83287	1.720458	171.3628	1.328721
6	10.69463	0.8455436	22.61096	1.403317	56.42005	1.640403	201.6754	1.339187
7	12.34051	0.8319845	26.05602	1.398832	64.92529	1.591137	231.6762	1.475725
8	13.98614	0.8219738	29.49789	1.384163	74.48263	1.471176	261.4938	1.476164
9	15.63165	0.8139109	32.95155	1.344156	82.13569	1.379199	290.7883	1.746614
10	17.27717	0.8057989	36.42348	1.265567	90.82749	1.270186	318.4558	1.919018
11	18.92296	0.7968315	39.90958	1.190464	99.45799	1.266908	344.8066	2.656835
12	20.56907	0.7864942	43.39833	1.120982	107.9627	1.266408	371.3062	3.662810
13	22.21551	0.7753367	46.87870	1.087722	116.3581	1.365504	398.4251	4.771676
14	23.86217	0.7639133	50.34671	1.066148	124.7126	1.426507	425.9647	5.364144
15	25.50891	0.7533244	53.80613	1.066216	133.0815	1.523635	453.9592	5.187028
16	27.15558	0.7437021	57.26454	1.053514	141.4751	1.560169	482.3039	4.937506
17	28.80213	0.7354858	60.72865	1.038746	149.8723	1.642241	511.0129	4.296126
18	30.44857	0.7278780	64.20089	1.004302	158.2538	1.730880	540.0888	3.907357
19	32.09501	0.7207162	67.67827	0.9771636	166.6222	1.890389	569.4771	3.334017
$Re = 100000$								
0	0.8527500	1.224706	2.057088	7.669958	4.378259	36.89763	8.669194	149.0541
1	2.569709	1.071055	6.416271	4.949062	15.56713	12.23460	52.45694	10.96869
2	4.284535	1.018010	10.91605	3.423840	27.82778	5.824700	98.66410	4.135897
3	6.000345	0.9737277	15.48217	2.577208	40.24260	3.801105	144.9288	2.623060
4	7.716726	0.9363319	20.02775	2.158581	52.41224	3.060748	189.2804	2.144607
5	9.432970	0.9074128	24.50747	1.988794	64.19979	2.806684	232.0528	2.031511
6	11.14861	0.8885302	28.92332	1.953850	75.68026	2.668047	273.4533	1.861865
7	12.86355	0.8780737	33.30667	1.978013	87.07153	2.548486	314.9421	1.836754
8	14.57798	0.8735705	37.69144	1.966780	98.57651	2.289103	356.9607	1.589301
9	16.29221	0.8716454	42.10002	1.903169	110.2565	2.086113	399.6086	1.689550
10	18.00650	0.8686807	46.53793	1.779837	122.0276	1.861885	441.9238	1.567629
11	19.72118	0.8630604	50.99263	1.675925	133.7528	1.813739	483.3555	1.943482
12	21.43633	0.8541790	55.44331	1.590364	145.3451	1.737024	523.4539	1.930795
13	23.15190	0.8435228	59.87590	1.566095	156.8185	1.794289	562.6655	2.479385
14	24.86765	0.8325389	64.29075	1.551598	168.2594	1.732525	601.2783	2.513732
15	26.58334	0.8234567	68.69899	1.557649	179.7472	1.750248	639.4575	3.152696
16	28.29877	0.8165827	73.11397	1.524803	191.2937	1.646895	677.1364	3.506139
17	30.01390	0.8123880	77.54388	1.483944	202.8414	1.675537	714.8141	4.576444
18	31.72881	0.8093270	81.98724	1.414376	214.3179	1.631505	752.8343	5.667472
19	33.44369	0.8065385	86.43420	1.371684	225.6981	1.752710	791.1887	6.676805

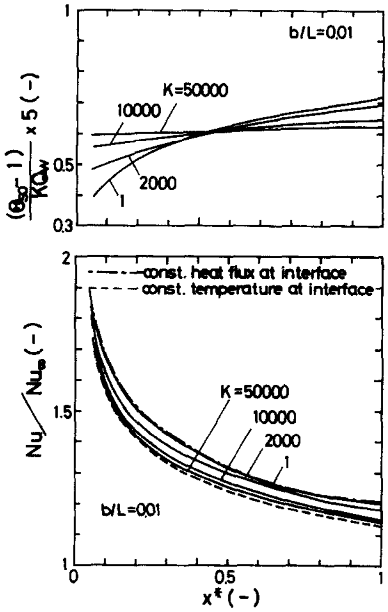

FIG. 3. Effect of ratios of conductivity of plate to that of fluid on interfacial temperature profiles and local Nusselt numbers (constant heat flux at lower surface of plate, $Pr = 0.7$, $Re = 10000$, $Nu_\infty = 25.9421$, $y_0/L = 0.02$).

examples evaluated for $Re = 10000$, $Pr = 0.7$, $y_0/L = 0.02$, $0.001 \leqslant b/L \leqslant 0.05$ and $1 \leqslant K \leqslant 50000$. Figure 3 shows the effect on the Nu/Nu_∞ and the interfacial temperature distribution for various conductivity ratios, K, for the constant heat flux at the lower plate surface and for $b/L = 0.01$. The effect of axial conduction in the plate on the heat transfer increases with increase of conductivity ratio K. With an increase of K, the

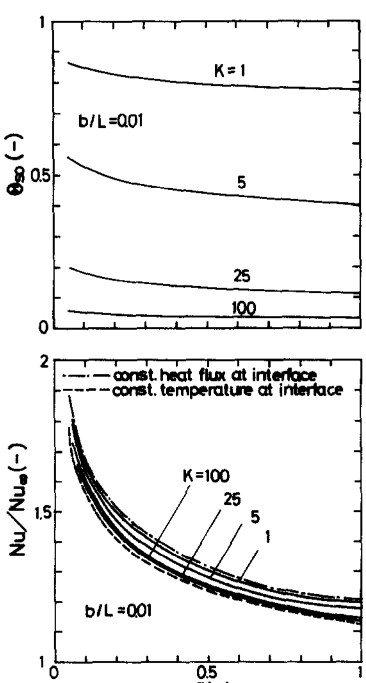

FIG. 4. Effect of ratios of conductivity of plate to that of fluid on interfacial temperature profiles and local Nusselt numbers (constant temperature at lower surface of plate, $Pr = 0.7$, $Re = 10000$, $Nu_\infty = 25.9421$, $y_0/L = 0.02$).

Nu/Nu_∞ and the interfacial temperature distribution approach asymptotically to the result for the constant temperature at the interface. At $K = 50000$, the Nu/Nu_∞ distribution almost agrees with the result for the constant interfacial temperature and the interfacial temperature distribution is almost uniform. On the other hand, for small K, the effect of axial conduction in the plate becomes insignificant and may be neglected. For $K \leqslant 1$, it is obvious that the Nu/Nu_∞ distribution agrees with the result for the constant heat flux at the interface. The effect on the interfacial temperature distribution for various conductivity ratios has a tendency to agree with that of Sell *et al.* [2].

Figure 4 shows results for the constant temperature at the lower plate surface and for $b/L = 0.01$. It is indicated that as K increases, the interfacial temperature profile approaches the limiting profile predicted for the constant interfacial temperature. For $K \geqslant 100$, the Nu/Nu_∞ distribution almost agrees with the result for the constant interfacial temperature. When K decreases to a small value, the temperature difference $(t_{so} - t_{fm})$ between the interfacial temperature and the mixed mean temperature is very small but the order of the temperature difference varies considerably with x^*. However, the temperature difference $(t_w - t_{so})$ between the lower plate surface and the interfacial temperature becomes almost uniform. Consequently, the heat flux at the interface becomes uniform. For $K \leqslant 1$, it seems that the Nu/Nu_∞ distribution, contrary to the above tendencies for large K, approximately agrees with the results for the constant heat flux at the interface.

Now, the effect of the flat plate thickness on the heat transfer is discussed. Figure 5 shows the results for the constant heat flux at the lower plate surface for $K = 10000$. It seems to be a quite reasonable con-

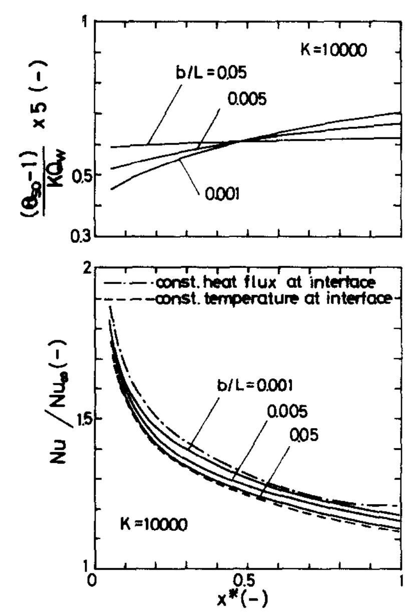

FIG. 5. Effect of plate thickness on interfacial temperature profiles and local Nusselt numbers (constant heat flux at lower surface of plate, $Pr = 0.7$, $Re = 10000$, $Nu_\infty = 25.9421$, $y_0/L = 0.02$).

clusion that for small b/L, the Nu/Nu_∞ distribution is in good agreement with the result for the constant heat flux at the interface. On the other hand, the effect of axial conduction in the plate increases for large b/L and the interfacial temperature distribution becomes almost uniform. The effect of b/L on the heat transfer varying with K tends to increase with increase of K. The effect of plate thickness on the interfacial temperature has a tendency to agree with that of Sell *et al.* [2] and Davis *et al.* [3]. Figure 6 shows the results for the constant temperature at the lower plate surface for $K = 1$. When b/L is very small, it can be treated as $\Theta_{so} = 0$, that is, $t_{so} = t_w$(const.). And, the solution approaches asymptotically to that of heat transfer to turbulent flow neglecting wall conduction. The effect of b/L on the heat transfer varying with K, contrary to the above tendencies for the constant heat flux at the lower plate surface, tends to increase with decrease of K.

The effects of axial conduction on the heat transfer obtained for various values of Reynolds numbers and Prandtl numbers have the similar tendencies as are illustrated in Figs. 3–6. The difference between the local Nusselt numbers calculated with and without the effect of wall conduction is markedly affected by Reynolds numbers and Prandtl numbers. Tables 2 and 3 are the results which show the degree of difference obtained for $10^4 \leqslant Re \leqslant 10^5$ and $0.01 \leqslant Pr \leqslant 0.7$. In Tables 2 and 3, the conditions of calculations are $y_0/L = 0.02$ and $b/L = 0.05$. Moreover, K is taken to be 50000 for the constant heat flux at the lower plate surface and 1 for the constant temperature at the lower plate surface. Nu_F shows the local Nusselt number based on the constant heat flux at the interface and Nu_T shows the local Nusselt number based on the constant interfacial temperature.

Table 2. Degree of difference between the local Nusselt numbers calculated with and without the effect of wall conduction in the range of $0.25 \leqslant x^* \leqslant 0.75$ (constant heat flux at lower surface of plate, $K = 50000$, $y_0/L = 0.02$, $b/L = 0.05$)

	$Pr = 0.01$	$Pr = 0.1$	$Pr = 0.7$
Re	$100(Nu_F - Nu)/Nu_F$ (%)		
10000	14–21	15–17	4–7
50000	23–27	13–14	4–6
100000	25–27	11–14	3–5

Table 3. Degree of difference between the local Nusselt numbers calculated with and without the effect of wall conduction in the range of $0.25 \leqslant x^* \leqslant 0.75$ (constant temperature at lower surface of plate, $K = 1$, $y_0/L = 0.02$, $b/L = 0.05$)

	$Pr = 0.01$	$Pr = 0.1$	$Pr = 0.7$
Re	$100(Nu - Nu_T)/Nu_T$ (%)		
10000	12–23	16–18	4–7
50000	26–34	15–16	4–7
100000	30–35	13–16	3–6

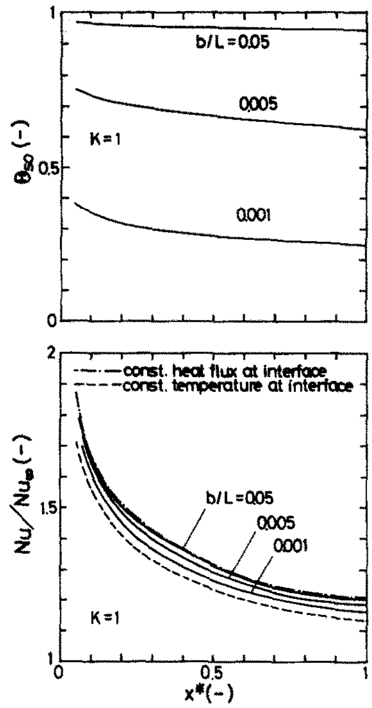

FIG. 6. Effect of plate thickness on interfacial temperature profiles and local Nusselt numbers (constant temperature at lower surface of plate, $Pr = 0.7$, $Re = 10000$, $Nu_\infty = 25.9421$, $y_0/L = 0.02$).

The local Nusselt numbers in Table 2 approach asymptotically to the result for Nu_T, and those in Table 3 approach asymptotically to the result for Nu_F as described previously. From the Tables 2 and 3, it is shown that the effect of wall conduction on the heat transfer can not be neglected for low Prandtl number. For $Pr = 0.01$, the degree of difference is about 12–35%. The degree of difference for $Pr = 0.1$ and 0.7 decreases with increase of Reynolds number, however, for $Pr = 0.01$ increases with increase of Reynolds number. This reason can estimate from Sleicher and Tribus paper [15] that showed the increase of difference between Nu_F and Nu_T with Reynolds number at very low Prandtl number. Though the degree of difference is not shown in Tables, for $Pr = 10$, it is less than 2–3% and the effect of wall conduction on the heat transfer is insignificant. In the range of low Prandtl number, it is obvious that in the design and analysis of heat exchange equipment, axial conduction in the wall can have a significant effect on the heat transfer and temperature field in the fluid adjacent to the wall. In the laminar flow, the effects on the conjugated heat transfer have similar tendencies, as in the turbulent flow. The degree of difference is about 20% and shows a few variations for Péclét number [14], but the effect of Prandtl number on laminar conjugated heat transfer is smaller than that on turbulent conjugated heat transfer.

This model can be applied to a more precise design of heat exchange equipment, and the interfacial temperature distribution that is predicted from this model will enable the adequate selection of insulation materials of a duct.

3. EXPERIMENTAL EXAMINATION

3.1. *Experimental apparatus and procedure*

Measurement of the local heat transfer coefficient was performed for a turbulent flow fully developed in the rectangular duct consisting of a 200 cm length, 15 cm wide and 3 cm high. A 30 cm heat-transfer plate, consisting of hard polyvinyl chloride extending the width of the duct, was installed about 150 cm from air inlet. A heating chamber, attached to the bottom of plate, was used as the heat source to the system. The upstream and downstream edges of the plate were insulated. The entire test section was also insulated by surrounding the styroform plates to minimize heat losses. A scheme of experimental apparatus is shown in Fig. 7.

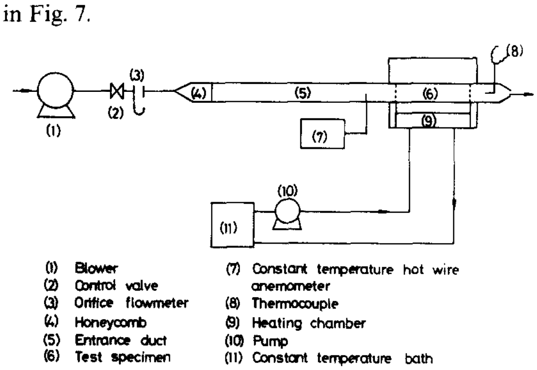

FIG. 7. Schematic diagram of experimental apparatus.

To obtain the interfacial temperature and the local heat flux, the thermocouples were installed in the plate at five positions along the flow and at five positions perpendicular to the direction of flow. From the measured temperature distribution in the plate, the interfacial temperature and the local heat flux at the interface were estimated and the local Nusselt number was calculated.

3.2. *Experimental results and considerations*

The experiments were made keeping a constant temperature at the lower surface of the plate. From the data of velocity, it was confirmed that the horizontal velocity distribution of the fluid in the duct was almost uniform within 10 cm wide. The vertical velocity distribution of the fluid at the center line of the duct shown in Fig. 8 fairly agrees with the results calculated by the method of Mizushina *et al.* [8]. Experimental

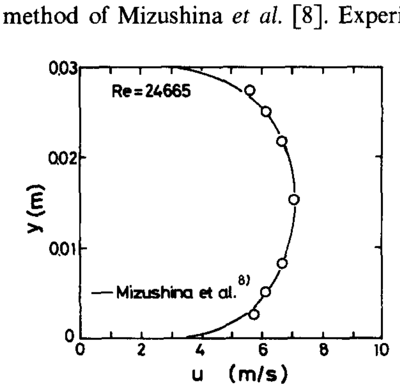

FIG. 8. Vertical velocity distribution of fluid at center line of duct.

data of Larson *et al.* [11] on the heat transfer between parallel plates are in good agreement with their numerical solution, although they observed the secondary flow that occurs in the corners of the duct. Therefore, it seems that the flow in the experimental duct may be considered to be two dimensional. Figure 9 shows the experimental data for unsymmetrical heat transfer in a 15 by 3 cm duct and numerical solution for the two dimensional channel flow. The predicted

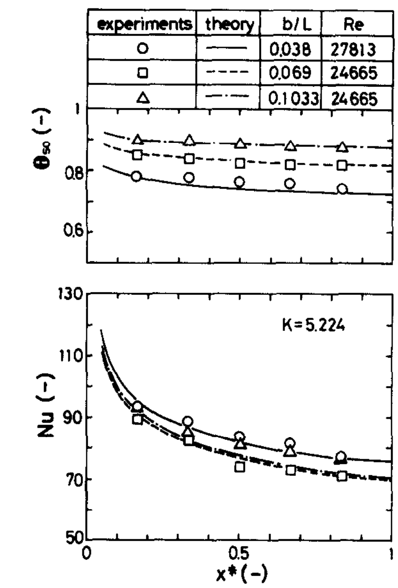

FIG. 9. Comparison of predicted numerical solutions with experimental data (constant temperature at lower surface of plate, $Pr = 0.7$, $y_0/L = 0.05$).

temperature distributions at the interface are in reasonably good agreement with the measured interfacial temperatures. Somewhat poorer agreement is obtained in comparison with the predicted and measured local Nusselt numbers, however, for the Nusselt numbers calculated from the experimental data are more subject to errors in the measurements than are the dimensionless temperature profiles. Consequently, it is considered that these experimental results are sufficiently accurate for technical purposes and that this theoretical model represents fairly well the heat-transfer phenomena.

CONCLUSIONS

By the theoretical analysis on the effect of axial conduction in the wall on the heat transfer with turbulent flow between parallel plates, it was found that the Prandtl number and the Reynolds number of the fluid, the ratio of conductivity of the wall to that of the fluid, $K = k_s/k_f$, and the thickness to length ratio of the wall, b/L, were the important parameters in determining the effects of the axial conduction. These effects are especially true in the low Prandtl number.

1. In the range of low Prandtl number, it is shown that the effect of wall conduction can have a significant effect on the heat transfer (the local Nusselt number and the interfacial temperature distribution).

2. For the constant heat flux at lower surface of the plate, the effect of wall conduction on the heat transfer increases with increase of conductivity ratio K.
3. For the constant temperature at lower surface of the plate, the effect of wall conduction on the heat transfer increases with decrease of conductivity ratio K.

It was confirmed, as expected, that the effect of wall conduction could be neglected reasonably when the plate was very thin. In the laminar flow, the effects on the conjugated heat transfer have similar tendencies, as in the turbulent flow. However, the effect of Prandtl number on laminar conjugated heat transfer is smaller than that on turbulent conjugated heat transfer.

Experiments were conducted with parallel flow for the constant temperature at the lower surface of the plate. The experimental results were in good agreement with the theoretical solutions. Therefore, it was found that the present model could be applied to the design of a heat exchanger and the selection of insulation materials of a duct.

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APPENDIX 1

Eddy viscosity and velocity distribution

$0 \leqslant y^+ \leqslant y_1^+$

$$\frac{\varepsilon_v}{v} = A(y^+)^3 \tag{1}$$

$$u^+ = \frac{1}{6A^{2/3}}\left(A^{1/3}+\frac{1}{y_0^+}\right)\times l_n\left|\frac{(y^+ + 1/A^{1/3})^3}{(y^+)^3+1/A}\right| + \frac{1}{3^{1/2}A^{2/3}}\left(A^{1/3}-\frac{1}{y_0^+}\right)\times\left[\tan^{-1}\left(\frac{2y^+ - 1/A^{1/3}}{3^{1/2}/A^{1/3}}\right)+\frac{\pi}{6}\right] \tag{2}$$

$y_1^+ \leqslant y^+ \leqslant y_2^+$

$$\frac{\varepsilon_v}{v} = 0.4y^+\left(1-\frac{y^+}{y_0^+}\right)-1 \tag{3}$$

$$u^+ = 2.5\ln y^+ + 5.5 \tag{4}$$

$y_2^+ \leqslant y^+ \leqslant y_0^+$

$$\frac{\varepsilon_v}{v} = 0.07y_0^+ \tag{5}$$

$$u^+ = \frac{y^+ - y_2^+}{1+0.07y_0^+}\left(1-\frac{y^+ + y_2^+}{2y_0^+}\right) + 2.5\ln y_2^+ + 5.5 \tag{6}$$

where A, y_1^+, y_2^+ are described by

$$A(y_1^+)^3 = 0.4y_1^+\left(1-\frac{y_1^+}{y_0^+}\right)-1 \tag{7}$$

$$\int_0^{y_1^+}\frac{1-y^+/y_0^+}{1+A(y^+)^3}\,\mathrm{d}y^+ = 2.5\ln y_1^+ + 5.5 \tag{8}$$

$$y_2^+ = \tfrac{1}{2}\{y_0^+ - [0.3(y_0^+)^2 - 10y_0^+]^{1/2}\}. \tag{9}$$

INFLUENCE DE LA CONDUCTION A L'INTERIEUR DE LA PAROI SUR LE TRANSFERT DE CHALEUR EN ECOULEMENT TURBULENT ENTRE PLAQUES PARALLELES

Résumé—Après avoir examiné expérimentalement l'influence de la conduction thermique à l'intérieur de la paroi sur le transfert de chaleur en écoulement turbulent établi, on a développé une étude analytique du transfert de chaleur avec conduction axiale dans la paroi limitant le fluide en écoulement turbulent, afin de déterminer l'influence de la conduction dans la paroi sur le transfert de chaleur en écoulement turbulent entre plaques planes parallèles.

A partir des résultats numériques pour des nombres de Reynolds compris dans le domaine $10^4 < Re < 10^5$ et des nombres de Prandtl dans le domaine $0{,}01 < Pr < 10$, il est confirmé que le rapport de la conductivité thermique de la paroi à celle du fluide, ainsi que l'épaisseur de la paroi, pouvaient avoir une influence significative sur le transfert de chaleur et le champ de température dans le fluide adjacent à la paroi. Des expériences sur plaque plane ont été réalisées et les résultats obtenus sont en bon accord avec les résultats analytiques.

DER EINFLUSS DER WANDWÄRMELEITUNG AUF DEN WÄRMEÜBERGANG BEI TURBULENTER STRÖMUNG ZWISCHEN PARALLELEN PLATTEN

Zusammenfassung—Es wird der Einfluß der Wandwärmeleitung auf den stationären Wärmeübergang bei turbulenter Strömung untersucht. Für den Wärmeübergang bei axialer Wärmeleitung in den Begrenzungswänden eines turbulent strömenden Fluides wird ein Modell vorgestellt, das die Möglichkeit bietet, den Einfluß der Wärmeleitung in den Wänden auf den Wärmeübergang bei turbulenter Strömung zwischen parallelen Platten zu bestimmen.

Die Ergebnisse der numerischen Rechnung für die Bereiche $10^4 \leqslant Re \leqslant 10^5$ und $0{,}01 \leqslant Pr \leqslant 10$ bestätigen, daß das Verhältnis der Wärmeleitfähigkeiten von Wand und Fluid sowie die Wanddicke einen beträchtlichen Einfluß auf den Wärmeübergang und die Temperatur der wandnahen Fluidschicht ausüben. Experimentelle Untersuchungen an ebenen Platten ergaben eine gute Übereinstimmung mit den analytischen Ergebnissen.

ВЛИЯНИЕ ТЕПЛОПРОВОДНОСТИ СТЕНКИ НА ТЕПЛООБМЕН ПРИ ТУРБУЛЕНТНОМ ТЕЧЕНИИ ЖИДКОСТИ МЕЖДУ ПАРАЛЛЕЛЬНЫМИ ПЛАСТИНАМИ

Аннотация — Исследуется влияние теплопроводности стенки на стационарный теплообмен с турбулентным потоком жидкости. Для оценки влияния теплопроводности стенки на теплообмен с турбулентным потоком жидкости проводится теоретический анализ процесса теплообмена на модели течения между параллельными пластинами при учете теплопроводности стенки, ограничивающей поток жидкости. Результаты численных расчетов при $10^4 \leqq Re \leqq 10^5$ и $0{,}01 \leqq Pr \leqq 10$ подтвердили существенное влияние величины отношения теплопроводности стенки к теплопроводности жидкости и толщины стенки на теплообмен и температурное поле жидкости вблизи стенки. Эксперименты проводились на плоской пластине. Получено хорошее соответствие между экспериментальными и расчетными данными.